Lecture 22 - Nov. 26

Recursion

Tracing: Factorial, Fibonacci Sequence
Recursion on Str.: Palindrome, Reversal
Exam Info

Announcements/Reminders

- **Lab5** released
  + Required study: **Abstract Classes** & **Interfaces**
- Learning resources on **Recursion**
- **Exam** Review Sessions eClass Polling

Recursive Solution: factorial

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1) \cdot (n-2) \cdot \cdots \cdot 3 \cdot 2 \cdot 1 & \text{if } n \geq 1 \end{cases}$$

not recursive

no recursive call to "!"

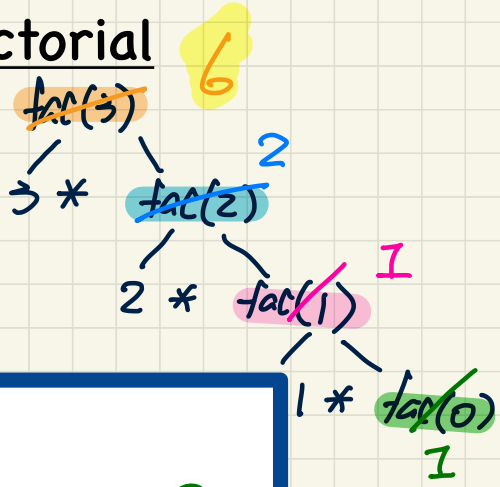
recursive call

$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n * (n-1)! & \text{if } n \geq 1 \end{cases}$$

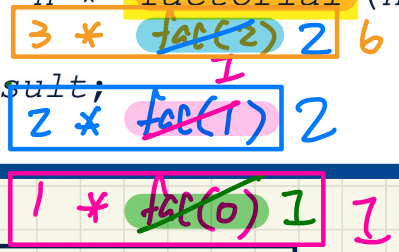
solution to a strictly smaller problem

Recursive Solution in Java: factorial

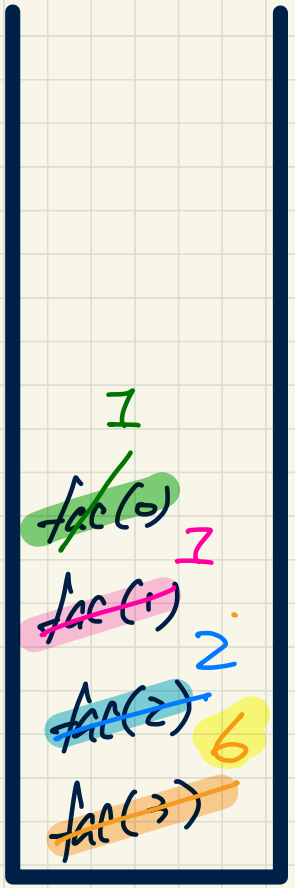
$$n! = \begin{cases} 1 & \text{if } n = 0 \\ n \cdot (n-1)! & \text{if } n \geq 1 \end{cases}$$




```
int factorial(int n){
  int result;
  if(n == 0) { /* base case */ result = 1; }
  else { /* recursive case */
    result = n * factorial(n - 1);
  }
  return result;
}
```



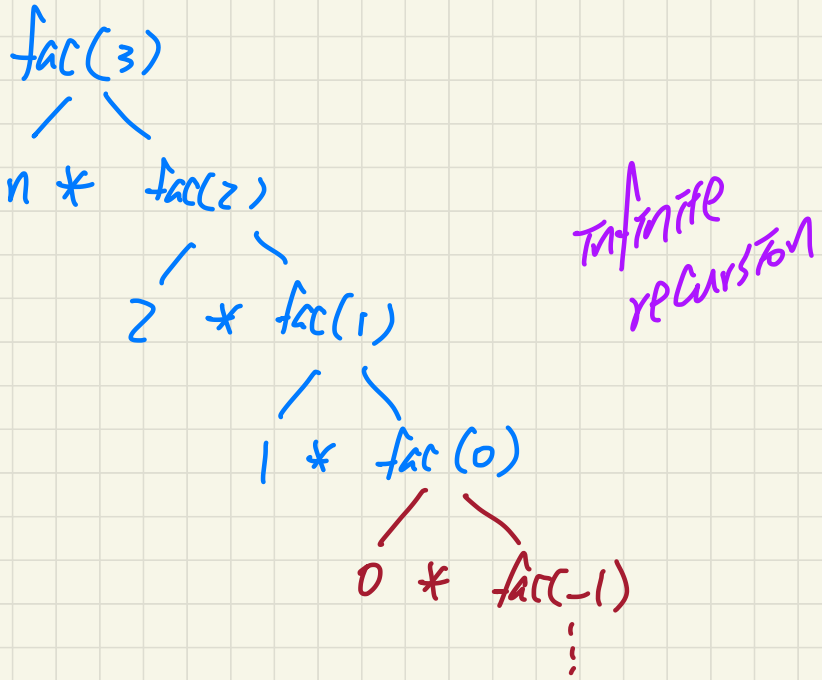
Example: factorial(3)



Runtime Stack

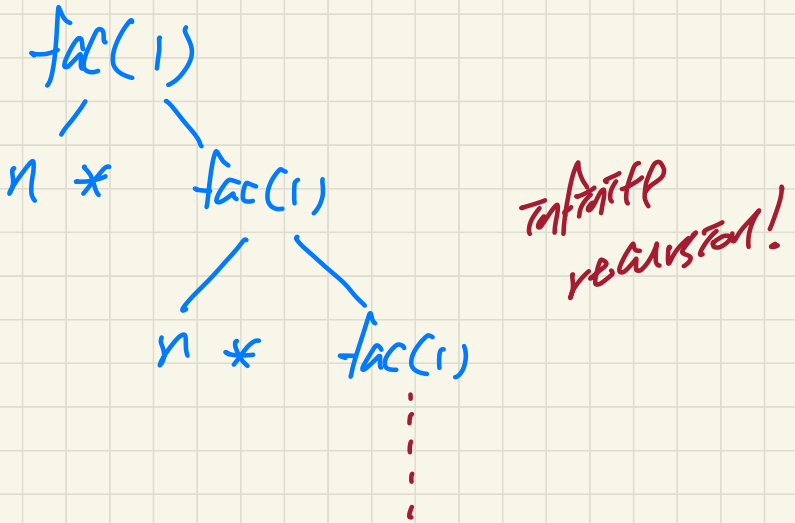
Common Errors of Recursion (1)

```
int factorial (int n) {
  return n * factorial (n - 1);
}
```



Common Errors of Recursion (2)

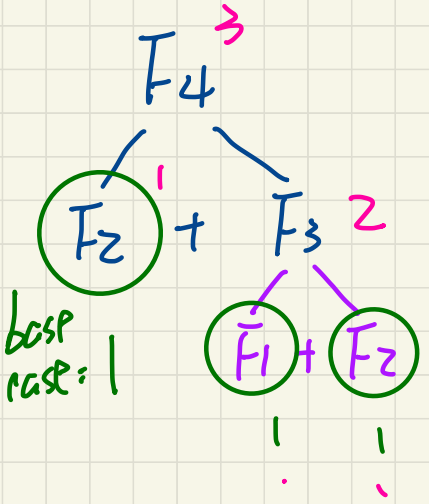
```
int factorial (int n) {
  if(n == 0) { /* base case */ return 1; }
  else { /* recursive case */ return n * factorial (n); }
}
```



Recursive Solution: Fibonacci Numbers

F = 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ...

F_1 F_2 F_4 F_6 F_7 F_8



$$F_n = \begin{cases} 1 & \text{if } n = 1 \\ 1 & \text{if } n = 2 \\ F_{n-1} + F_{n-2} & \text{if } n \geq 3 \end{cases}$$

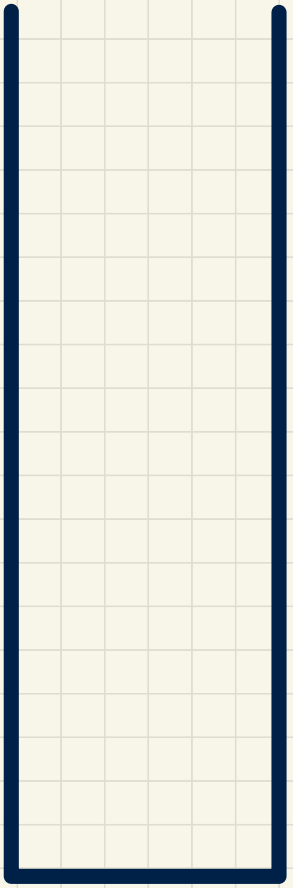
two recursive calls

Recursive Solution in Java: Fibonacci Numbers

$$F_n = \begin{cases} 1 & \text{if } n = 1 \\ 1 & \text{if } n = 2 \\ F_{n-1} + F_{n-2} & \text{if } n > 2 \end{cases}$$

```
int fib(int n) {
  int result;
  if(n == 1) { /* base case */ result = 1; }
  else if(n == 2) { /* base case */ result = 1; }
  else { /* recursive case */
    result = fib(n - 1) + fib(n - 2);
  }
  return result;
}
```

Example: fib(4)

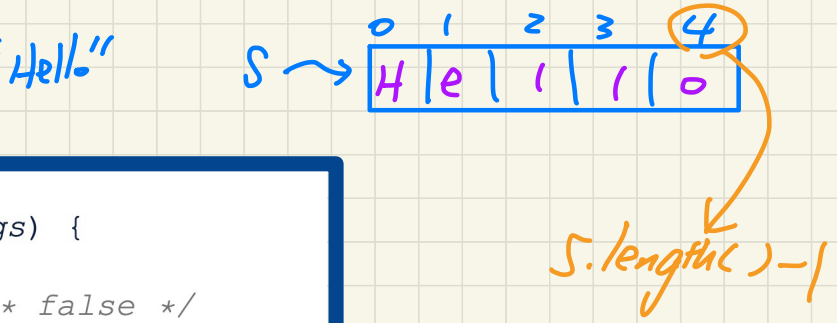


Runtime Stack

Use of String

```
public class StringTester {
  public static void main(String[] args) {
    String s = "abcd";
    System.out.println(s.isEmpty()); /* false */
    /* Characters in index range [0, 0) */
    String t0 = s.substring(0, 0);
    System.out.println(t0); /* "" */
    /* Characters in index range [0, 4) */
    String t1 = s.substring(0, 4);
    System.out.println(t1); /* "abcd" */
    /* Characters in index range [1, 3) */
    String t2 = s.substring(1, 3);
    System.out.println(t2); /* "bc" */
    String t3 = s.substring(0, 2) + s.substring(2, 4);
    System.out.println(s.equals(t3)); /* true */
    for(int i = 0; i < s.length(); i ++) {
      System.out.print(s.charAt(i));
    }
    System.out.println();
  }
}
```

s = "Hello"



0 1 2 3

[0, -1] → i > j → ""

[0, 3]

[1, 2]

s.substring(i, j)

form a substring by collecting characters

in: [i, j)

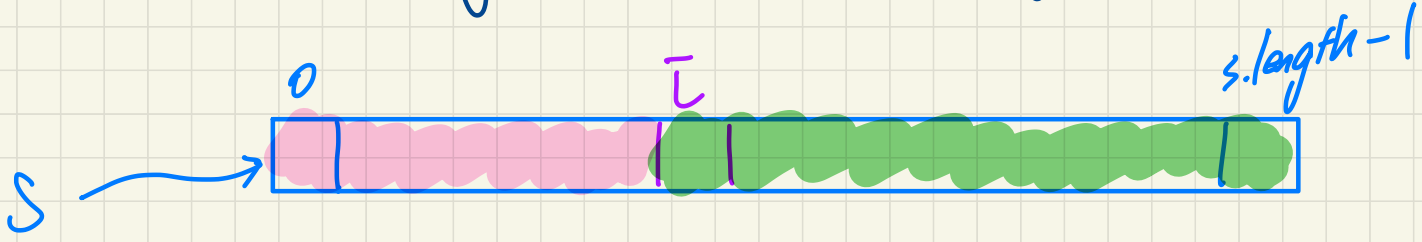
typically: i ≤ j

[i, j-1]

For any non-null string s,

given int. i s.t. $0 \leq i \leq s.length - 1$

$s.equals(s.substring(0, i) + s.substring(i, s.length()))$



Recursions on Strings

isP("racecar")

↳ r == r && isP("aceca")

Palindrome

"racecar"

"aracecars"

"raceacar"

Reversal

"abcd"

Number of Occurrences

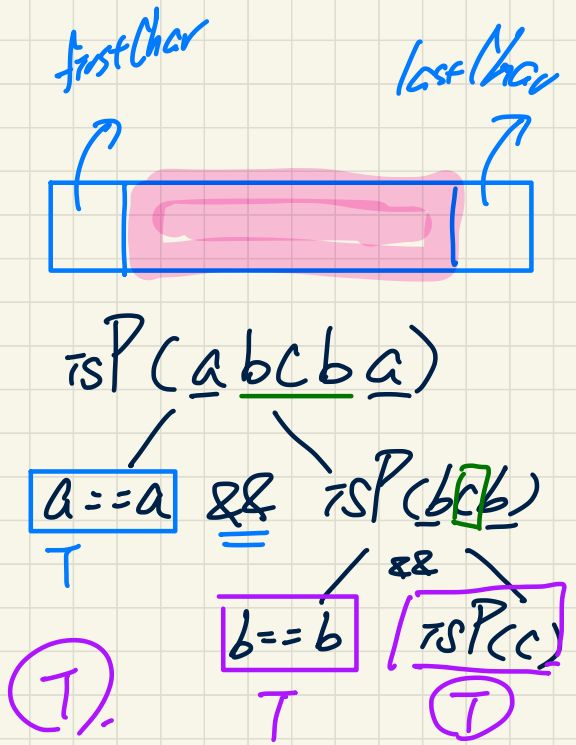
"abca"

'a'

'b'

Problem: Palindrome

```
boolean isPalindrome (String word) {
  if(word.length() == 0 || word.length() == 1) {
    /* base case */
    return true;
  }
  else {
    /* recursive case */
    char firstChar = word.charAt(0);
    char lastChar = word.charAt(word.length() - 1);
    String middle = word.substring(1, word.length() - 1);
    return
        firstChar == lastChar
        /* See the API of java.lang.String.substring. */
        && isPalindrome (middle);
  }
}
```



ex. isP(racecar)

ex. isP(racebar)

Exam Info

- When: 10am to 1pm, Sunday, December 8
- Where: TC Sobeys
- Format: Some Multiple Choice & Mostly Written
- Coverage: **Everything** (lecture materials & labs)
  + slides, iPad notes, code examples
  + https://codingbat.com/java/Recursion-1
- Restrictions:
  + No data sheet
  + No sketch paper (Exam booklet includes it)
- What you should bring:
  + Valid, Physical Photo ID (strict)
  + Water/Snack